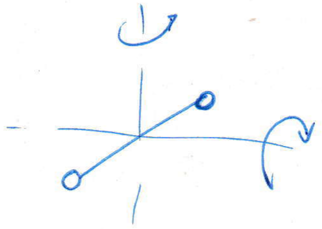
Zasada ekwipartycji energii

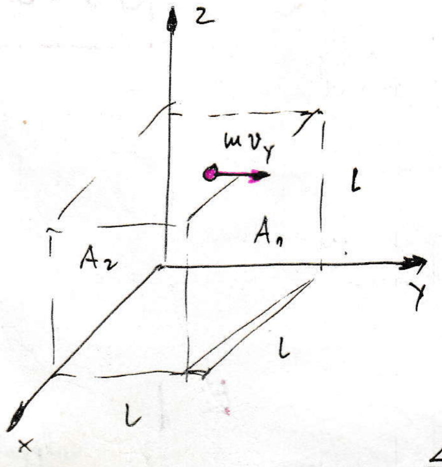
$E_v = \frac{1}{2} k_B T$ $\longrightarrow$ na każdy stopień swobody $\Big\{$ bo $\frac{1}{2} m \overline{v^2} = \frac{3}{2} k_B T$

tj. $U = \frac{3}{2} k_B T$ (dla atomów)

$U = \frac{5}{2} k_B T$ (dla cząsteczek 2-atomowych)



Obliczanie ciśnienia z teorii kinetycznej.



$$\Delta p = p_k - p_p$$

$$p_p = m v_y \qquad p_k = -m v_y$$

$$\Delta p = -m v_y - m v_y = -2 m v_y$$

czas przelotu: $\frac{L}{v_y}$ (bez zderzeń

ilość zderzeń na jedn. czasu ze ścianką A_1 $= \frac{v_y}{2L}$

$$\frac{\Delta p}{\Delta t} = 2 m v_y \frac{v_y}{2L} = \frac{m v_y^2}{L}$$

jest n-cząstek stąd $F_{całk} = \frac{\Delta p_{całk}}{\Delta t} = \frac{m}{L}\left(v_{y_1}^2 + v_{y_2}^2 + \dots\right)$

$$p = \frac{F_{całk}}{A} = \frac{F_{całk}}{L^2} = \frac{m}{L^3}\left(v_{y_1}^2 + v_{y_2}^2 + \dots\right)$$

N – całk. liczba cząstek w naczyniu

$n_v = \frac{N}{V} = \frac{N}{L^3}$ (w jedn. objętości) $\Big\} \Longrightarrow p = m n_v \left(\frac{v_{y_1}^2 + v_{y_2}^2 + \dots}{N}\right)$

$$\overline{v_y^2} = \frac{v_{y_1}^2 + v_{y_2}^2 + \dots + v_{y_N}^2}{N}$$ (średnia prędkość kwadratowa)

$\boxed{p = \rho \overline{v_y^2}}$ $\qquad v^2 = v_x^2 + v_y^2 + v_z^2 \Rightarrow \overline{v_y^2} = \frac{1}{3}\overline{v^2}$

$\boxed{p = \frac{1}{3}\rho \overline{v^2}}$ $\qquad \Rightarrow \overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$

$$v_{śr. kw.} = \sqrt{\overline{v^2}} = \sqrt{\frac{3p}{\rho}}$$

np. dl. H_2 $\quad p = 1$ atm, $T = 0°C$

$v_{śr. kw} = 1840 \frac{m}{s}$

Interpretacja kinetyczna temperatury

$$pV = \frac{1}{3}\rho V \overline{v^2} = \frac{1}{3} m \overline{v^2} = \frac{1}{3} n\mu \overline{v^2}$$

$$E_k = \frac{1}{2} m v_1^2 + \frac{1}{2} m v_2^2 + \dots = \frac{1}{2} N m \frac{v_1^2 + v_2^2 + \dots + v_N^2}{N} = \frac{1}{2} N m \overline{v^2} = \frac{1}{2} n\mu \overline{v^2}$$

stąd $\frac{1}{3} n\mu \overline{v^2} = \frac{2}{3} E_k$ $\Longrightarrow$ $pV = \frac{2}{3} E_k$

$pV = nRT$

$\Downarrow$

$$\frac{2}{3} E_k = nRT$$

$$E_k = \frac{3}{2} nRT$$

$$\frac{1}{2} n\mu \overline{v^2} = \frac{3}{2} nRT \Rightarrow \boxed{\frac{1}{2}\mu \overline{v^2} = \frac{3}{2} RT}$$

całkowita energia ruchu postępowego mola gazu jest proporcjonalna do temperatury.

na cząsteczkę energia $\frac{1}{2}\left(\frac{\mu}{N_A}\right)\overline{v^2} = \frac{3}{2}\left(\frac{R}{N_A}\right)T$

$$\boxed{\frac{1}{2} m \overline{v^2} = \frac{3}{2} k_B T}$$

k_B – stała Boltzmanna

$$k_B = 1.38 \cdot 10^{23} \frac{J}{K}$$

Ciepło właściwe

$$U = \frac{3}{2} N_A k_B T = \frac{3}{2} RT \quad \text{(w skali Kelvina)}$$

C_p – ciepło molowe przy stałym ciśnieniu

C_v – " " " " objętości

$$dU = dQ - p\,dV$$

wykładnik adiabatyczny

$$\gamma = \frac{C_p}{C_v} = \frac{f+2}{f}$$

$$C_v = \frac{dQ}{dT} = \frac{dU + p\,dV}{dT}\quad (p\,dV = 0) = \frac{dU}{dT}$$

$$C_v = \frac{dU}{dT} = \frac{d}{dT}\left(\frac{3}{2}RT\right) = \frac{3}{2}R$$

$$dQ = C_v\,dT + p\,dV$$

$$\left(\frac{dQ}{dT}\right)_p = C_p$$

$$C_p\,dT = C_v\,dT + p\,dV$$

$$\left.\begin{aligned} pV &= RT \\ p\,dV &= R\,dT \end{aligned}\right\}$$

$$C_p = C_v + R$$

$$C_p - C_v = R$$

dla procesu izobarycznego

Wykazać, że dla przemiany adiabatycznej

$pV^{\varkappa} = \text{const}$ $\varkappa = \frac{C_p}{C_v}$

$$\underset{0}{dQ} = dU + p\,dV \qquad pV = RT \qquad d(pV) = R\,dT$$

$$C_v\,dT = -p\,dV \qquad p = \frac{RT}{V}$$

$$dT = -\frac{p\,dV}{C_v} = -\frac{RT\,dV}{C_v V}$$

$$\frac{dT}{T} = -\frac{R}{C_v}\frac{dV}{V} = -\frac{(C_p - C_v)}{C_v}\frac{dV}{V}$$

$$dQ = dU + p\,dV \qquad d(pV) = d(RT)$$

$$V\,dp + p\,dV = R\,dT \Rightarrow dT = \frac{V\,dp + p\,dV}{R}$$

$$c_V\,dT = -p\,dV \qquad -p\,dV = V\,dp - R\,dT$$

$$c_V\,dT = V\,dp - R\,dT$$

$$\cancel{c_V\,dT} = V\,dp - C_p\,dT + \cancel{c_V\,dT} \qquad \cancel{C_p\,dT} = \text{ bo } C_p = C_V + R$$

$$C_V \frac{1}{R}(V\,dp + p\,dV) = -p\,dV$$

$$V\,dp + p\,dV = -\frac{(C_p - C_V)}{C_V}\,p\,dV$$

$$V\,dp = -p\,dV\left(\frac{C_p - C_V + C_V}{C_V}\right) = -\varkappa\,p\,dV \qquad /:pV$$

$$\int\frac{dp}{p} = -\varkappa\int\frac{dV}{V} \Rightarrow \ln p = -\varkappa \ln V + C_1$$

$$\ln\left(pV^{\varkappa}\right) = C_1$$

$$\boxed{pV^{\varkappa} = \text{const}}$$

$$U = \frac{f}{2}k_B T$$

$$C_p = C_V + R = \frac{3}{2}R + R = \frac{5}{2}R$$

$$\varkappa = \frac{C_p}{C_V} = \frac{\frac{5}{2}}{\frac{3}{2}} = \frac{5}{3} = 1.67 \quad (\text{jednoatomowy gaz})$$

		$\varkappa$
mono	He	1.67
	A	1.67
duo	H_2	1.41
	O_2	1.40
	Cl_2	1.35
poly	CO_2	1.30
	SO_2	1.29
	NH_3	1.31

$$U = 3n\left(\tfrac{1}{2}RT\right) + 2n\left(\tfrac{1}{2}RT\right) = \frac{5}{2}nRT$$

$$C_V = \frac{5}{2}R \qquad C_p = \frac{7}{2}R \qquad \varkappa = \frac{7}{5} = 1.40$$

$$c_V = 3R \qquad c_p = 4R \Rightarrow c_p/c_V = \frac{4}{3} = 1.33$$

p. Dulonga-Petita bo $\boxed{U = 3n\left(\tfrac{1}{2}RT\right) + 3n\left(\tfrac{1}{2}RT\right) = 3nRT}$

liczba Avogadro $N_A = 6{,}022 \times 10^{23}\ \frac{\text{cząst}}{\text{mol}}$

objętość molowa $V_m = 22{,}41\ \text{dm}^3$

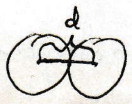
liczba Loschmidta $L = \frac{N_A}{V_m} = 2{,}68 \times 10^{19}\ \frac{\text{cz}}{\text{cm}^3}$

"procesy stochastyczne"
Marian Smoluchowski + Einstein
"fluktuacje gęstości"

Zderzenia a ruchy Browna

liczba zderzeń /s $\quad \left\{\frac{v}{\lambda} \sim 5\cdot10^{9}\ \frac{1}{s}\right\}$! $\quad d \sim 2\times10^{-8}\ \text{cm} \quad n_v \sim 3\cdot10^{19}\ \frac{1}{\text{cm}^3} \quad v \sim 10^{5}\ \frac{\text{cm}}{s}$

$\bar{\lambda}$ – średnia droga swobodna



$$\bar{\lambda} = \frac{vt}{\pi d^2 n_v v t} = \frac{1}{\pi d^2 n_v} \longrightarrow \frac{1}{\sqrt{2}\,\pi d^2 n_v} \qquad \bar{\lambda} \sim 2\cdot10^{-5}\ \text{cm}$$