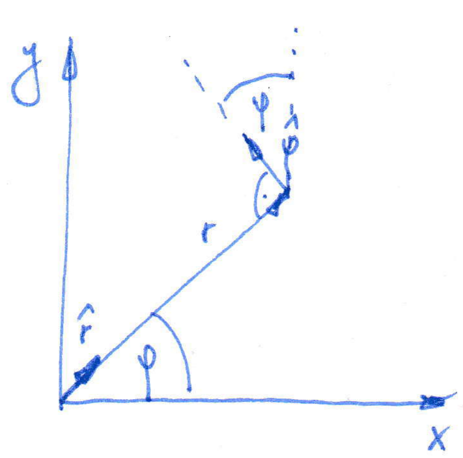
Układ biegunowy

Okrąg

$$\begin{cases} \text{środek } (r_0, \varphi_0) \\ \text{promień } a \end{cases}$$

$$r^2 - 2 r r_0 \cos(\varphi - \varphi_0) + r_0^2 = a^2$$



$$\hat{r} = (\cos\varphi, \sin\varphi)$$

$$\hat{\varphi} = (-\sin\varphi, \cos\varphi)$$

$$\vec{r} = r\hat{r}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(r\hat{r}) = \frac{dr}{dt}\hat{r} + r\frac{d\hat{r}}{dt}$$

$$\begin{cases} \dfrac{d\hat{r}}{dt} = (-\sin\varphi, \cos\varphi)\dfrac{d\varphi}{dt} = \dfrac{d\varphi}{dt}\hat{\varphi} \\ \dfrac{d\hat{\varphi}}{dt} = (-\cos\varphi, -\sin\varphi)\dfrac{d\varphi}{dt} = -\dfrac{d\varphi}{dt}\hat{r} \end{cases}$$

Prędkość:

$$\frac{d\vec{v}}{dt} = \frac{dr}{dt}\hat{r} + r\frac{d\varphi}{dt}\hat{\varphi} = v_r\hat{r} + v_\varphi\hat{\varphi} = (v_r, v_\varphi)$$

$$v_r = \frac{dr}{dt} = \dot{r} \qquad\qquad v_\varphi = r\frac{d\varphi}{dt} = r\dot{\varphi}$$

prędkość radialna — prędkość ~~transwersalna~~ kątowa (poprzeczna)

Przyspieszenie:

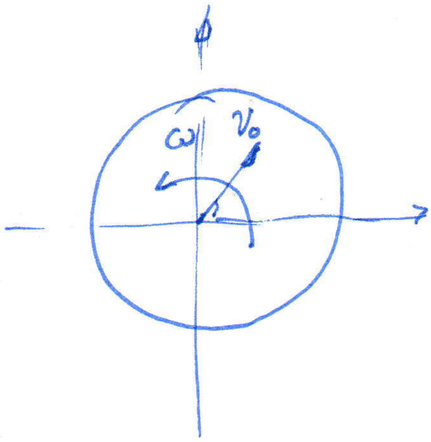
$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left[\frac{dr}{dt}\hat{r} + r\frac{d\varphi}{dt}\hat{\varphi}\right] = \frac{d^2r}{dt^2}\hat{r} + \frac{dr}{dt}\frac{d\hat{r}}{dt} +$$

$$+ \frac{dr}{dt}\frac{d\varphi}{dt}\hat{\varphi} + \frac{dr}{dt}\frac{d^2\varphi}{dt^2}\hat{\varphi} + \frac{dr}{dt}\frac{d\varphi}{dt}\frac{d\hat{\varphi}}{dt} =$$

$$\frac{d^2r}{dt^2}\hat{r} + \cancel{\frac{dr}{dt}\frac{d\varphi}{dt}\hat{\varphi}} + 2\frac{dr}{dt}\frac{d\varphi}{dt}\hat{\varphi} + \frac{dr}{dt}\frac{d^2\varphi}{dt^2}\hat{\varphi} - \frac{dr}{dt}\left(\frac{d\varphi}{dt}\right)^2\hat{r} =$$

$$\underbrace{\left[\frac{d^2r}{dt^2} - \frac{dr}{dt}\left(\frac{d\varphi}{dt}\right)^2\right]}_{a_r}\hat{r} + \underbrace{\left[\frac{dr}{dt}\frac{d^2\varphi}{dt^2} + 2\frac{dr}{dt}\frac{d\varphi}{dt}\right]}_{a_\varphi}\hat{\varphi}$$

$$a_r = \ddot{r} - \dot{r}\dot{\varphi}^2 \qquad\qquad a_\varphi = \dot{r}\ddot{\varphi} + 2\dot{r}\dot{\varphi}$$

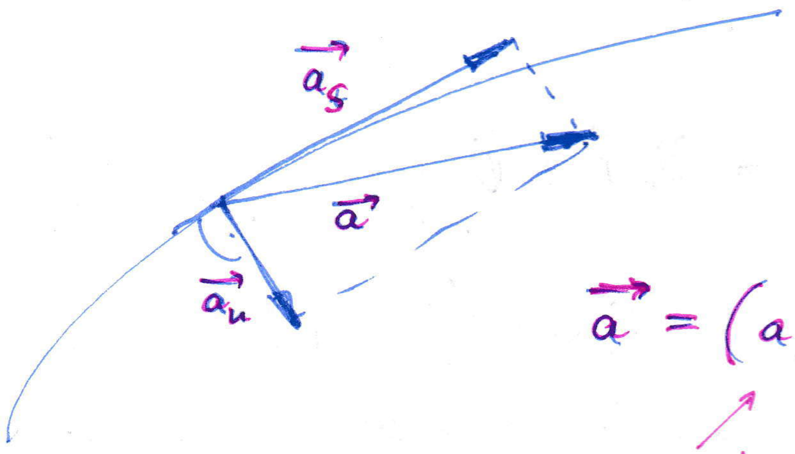


$$\begin{cases} r = v_0 t \\ \varphi = \omega t \end{cases}$$

$$\begin{cases} v_r = \dfrac{dr}{dt} = v_0 \\ v_\varphi = r\dfrac{d\varphi}{dt} = v_0 t \omega \end{cases}$$

$$\begin{cases} a_r = -\cancel{v_0}\,\omega^2 = -v_0\omega^2 t \\ a_\varphi = 2 v_0 \omega \end{cases}$$

$$a = \sqrt{a_r^2 + a_\varphi^2} = v_0\sqrt{\omega^4 t^2 + 4\omega^2} = v_0\omega\sqrt{\omega^2 t^2 + 4}$$



$$a = \sqrt{a_s^2 + a_n^2}$$

$$a_t = \frac{dv}{dt}$$

$$\vec{a} = (a_n, a_s)$$

normalne styczne