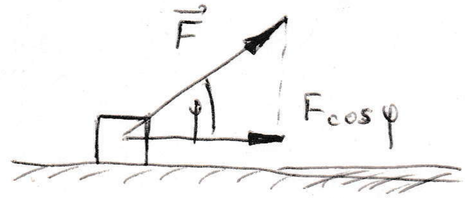
Praca i energia

$$\vec{a} = \frac{\vec{F}}{m} \qquad v = v_0 + at \qquad x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$W = \vec{F}\cdot\vec{r} = F r \cos\varphi$$

$$1\,J = 1\,N \cdot 1\,m$$

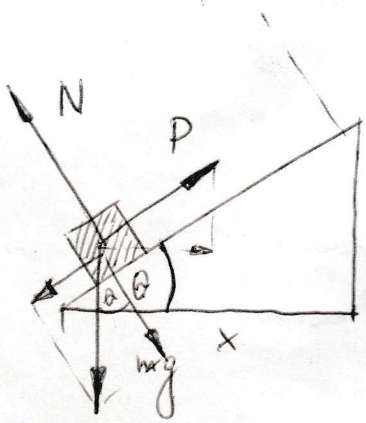
$$1\,eV = 1.602 \times 10^{-19}\,J$$



dla $\vec{F} = const$

$$\varphi = \frac{\pi}{2}\ (90^\circ) \Rightarrow W = 0$$

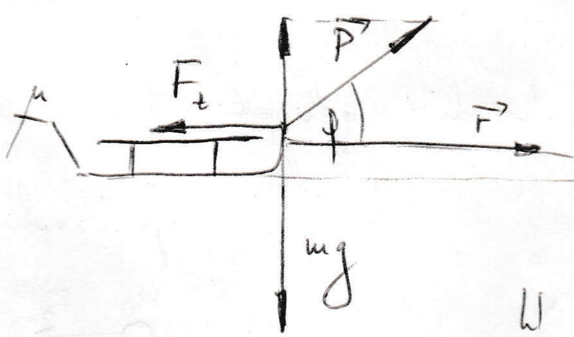
$$\varphi = 0 \Rightarrow W = max$$



$$W = \vec{P}\cdot\vec{d} =$$

$$P = mg\sin\theta - \mu mg\cos\theta$$

$$P = mg(\sin\theta - \mu\cos\theta)$$



$$W = \vec{P}\cdot\vec{r} = Pr\cos\varphi$$

$$P\cos\varphi - F_t = 0$$

$$F_t = \mu N = \mu mg$$

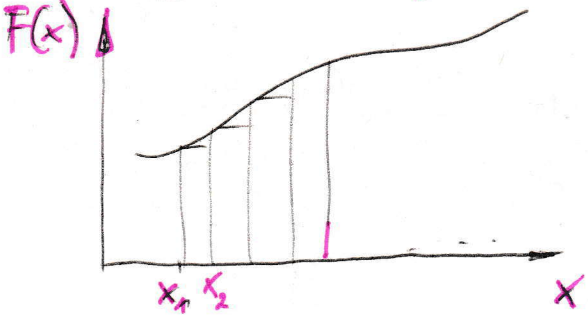
$$P\sin\varphi + N = mg$$

$$P = \mu mg \frac{1}{(\cos\varphi + \mu\sin\varphi)}$$

$$W = Pr\cos\varphi = \mu mg \frac{r\cos\varphi}{(\cos\varphi + \mu\sin\varphi)} = \mu mgr \frac{1}{(1 + \mu\,tg\varphi)}$$

$$\varphi = 0 \Rightarrow W = \mu mgr \ (max)$$

$$W = \int \vec{F} \cdot d\vec{r} = \int (F_x\,dx + F_y\,dy + F_z\,dz)$$



1-d

$$W = \int F(x)\,dx$$

$$W_{12} = \lim_{\Delta x_i \to 0} \sum_{i=1}^{N} F(x_i)\,\Delta x_i = \int_{x_1}^{x_2} F(x)\,dx$$

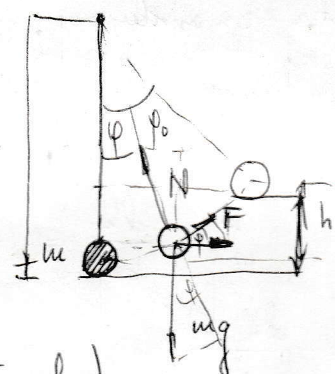
Sprężyna

$$F = -kx \Rightarrow \cancel{W(d) = \int_{x_1}^{x_2} (-kx)\,dx =}$$

by wykonać pracę należy przyłożyć siłę $F' = kx$ (przeciwną do $F = -kx$

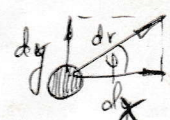
$$W = \int kx\,dx = k\,\frac{1}{2}x^2\Big|_1^2 = \frac{1}{2}k(x_2^2 - x_1^2)$$

Przypadek 2-d

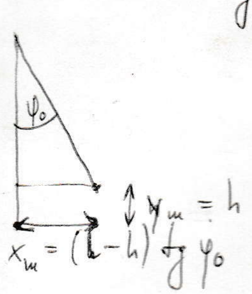


$$W = \int \vec{F} \cdot d\vec{r}$$

$$W = \int F \cos\varphi\,dr$$



$$W = \int_{x=0,\,y=0}^{x=L-h\,tg\,\varphi_0,\;y=h} (F_x\,dx + F_y\,dy) =$$



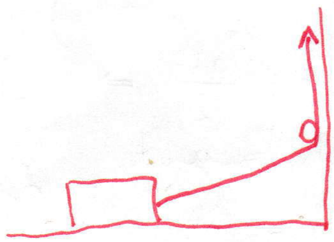
$$F_x = F = N \sin\varphi = mg\,tg\,\varphi$$

ale $\cancel{N = mg\cos\varphi \Rightarrow N =}$

$$N\cos\varphi = mg \Rightarrow N = \frac{mg}{\cos\varphi}$$

$$W = \int (mg\,tg\,\varphi\,dx + 0\,dy) = \int_0^{x_m} mg\,tg\,\varphi\,dx = mg\int_0^{y_m} dy = \underline{mg\,h}\,!$$

$$\left\{\; tg\,\varphi = \frac{dy}{dx} \Rightarrow tg\,\varphi\,dx = dy \right.$$



Twierdzenie o pracy i energii

(work) $W = \int \vec{F} \cdot d\vec{r}$ $\qquad \vec{F} = m\vec{a}$

$$W = m\int \vec{a}\cdot d\vec{r} = m\int \frac{d\vec{v}}{dt}\cdot d\vec{r} = m\int d\vec{v}\cdot\frac{d\vec{r}}{dt} = m\int d\vec{v}\cdot\vec{v} =$$

$$= m\int \vec{v}\cdot d\vec{v} = m\int_0 (v_x\, dv_x + v_y\, dv_y +) = m\frac{1}{2}(v_x^2 + v_y^2 + v_z^2)$$

$$= \frac{mv^2}{2} - \frac{mv_0^2}{2}$$

$$\boxed{W = \Delta E_k}$$

Praca wykonana przez siłę wypadkową F działającą na punkt materialny jest równa zmianie energii kinetycznej tego punktu.

Energia kinetyczna ciała znajdującego się w ruchu równa się pracy, jaką może wykonać to ciało, zanim się zatrzyma.

Moc

$$P = \frac{dW}{dt} = \frac{F\cdot d\vec{r}}{dt} = \vec{F}\cdot\vec{v}$$

(power) $\bar{P} = \frac{W}{t}$ $\qquad P(t) = \frac{dW(t)}{dt}$ $\qquad \left[1\,W = 1\,\frac{J}{s}\right]$

$$1\,W = \frac{kg\cdot\frac{m}{s^2}\cdot m}{s} = kg\,\frac{m^2}{s^3}$$

$$1\,KM = 736\,W \sim \frac{3}{4}\,kW$$

1 kWh = pracy jaką wykona urządzenie o mocy 1 kW w czasie 1h.

Praca w ruchu jednostajnie przyspieszonym

w ciągu t_k osiąga prędkość v_k

$$W = \int F\,dx = \int m\frac{dv}{dt}\,\cancel{dx} = \int m\,v\,dv = \int m\,v\,t\,\cancel{dx} =$$

$a = vt \qquad = \int m$

$$\int m\frac{v_k}{t_k}\,dx = \int_0^t m\left(\frac{v_k}{t_k}\right)^2 t\,dt = m\left(\frac{v_k}{t_k}\right)^2\frac{t^2}{2}$$

$$x = \frac{at^2}{2} \Rightarrow \frac{2x}{a} = t^2$$

$dx = a\,dt \qquad dx = \frac{v_k}{t_k}\,t\,dt$

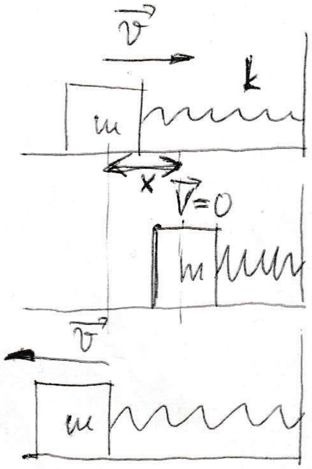
$a = \frac{v_k}{t_k}$

Zasada zachowania energii

$$W = \Delta E_k$$

n – sił $\vec{F}_1, \vec{F}_2, \dots \vec{F}_n \Longrightarrow W = W_1 + W_2 + \dots + W_n$

$$W_n = \int \vec{F}_n \cdot d\vec{s}$$

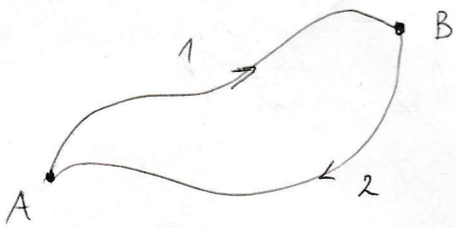


$$F = -kx$$

$$\int F\,dx = \int_0^x kx\,dx = \frac{1}{2}kx^2$$

Siły zachowawcze

Praca nie zależy od drogi



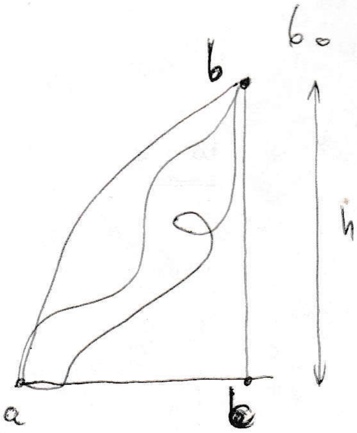
$$W^{(1)}_{ab} + W^{(2)}_{ba} = 0$$

$$W^{(1)}_{ab} = -W^{(2)}_{ba}$$

innymi słowy

$$\oint \vec{F} \cdot d\vec{r} = 0$$

$$\nabla \times \vec{F} = 0$$



$$W_{ab} = -mgh$$

Energia potencjalna

$U = E_p$

1D

$$\vec{F} = -\vec{\nabla} U_p \quad \Rightarrow \quad F(x) = -\frac{d}{dx}U(x)$$

$$\Delta U = -W = -\int_{x_0}^{x} F(x)\,dx$$

ważne!

$U = E_p$

$$\Delta U + \Delta E_k = 0$$

sprężyna

$\Downarrow$

$$E_p + E_k = \text{const}$$

zasada zach. energii

$$W = \Delta E_k = -\Delta E_p \quad \Rightarrow \quad W = -\Delta E_p$$

$$\Delta E_p = -W = -\int_{x_0}^{x} F(x)\,dx$$

$\Downarrow$

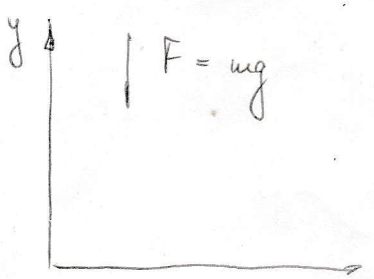
$$\frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv_0^2 + U(x_0)$$

zasada zach. energii mechanicznej

E_p ma sens dla sił zachowawczych

$$F(x) = -\frac{dU(x)}{dx}$$

$$\cancel{E_p}\; U(x) = -\int_{x_0}^{x} F(x)\,dx + U(x_0)$$



$$U(x) = -\int_{0}^{h} (-mg)\,dy + U(0) = mgy$$

$$\frac{1}{2}\cancel{m}v^2 + \cancel{m}gy = \frac{1}{2}\cancel{m}v_0^2$$

$$v^2 = v_0^2 - 2gy$$