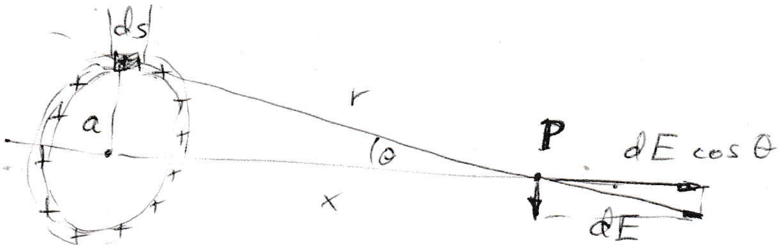
## Przykłady c.d.



$$dq = \frac{q}{2\pi a}\,ds$$

$$\vec{E} = \int d\vec{E}$$

$$E = \int dE\cos\theta \iff dE = \frac{dq}{4\pi\varepsilon_0 r^2} = \frac{q\,ds}{4\pi\varepsilon_0\, 2\pi a\,(a^2+x^2)}$$

$$E = \int_S \frac{x}{\sqrt{a^2+x^2}}\,\frac{q\,ds}{2\pi a\,(a^2+x^2)\,4\pi\varepsilon_0} = \frac{x\,q\,\cancel{2\pi a}}{4\pi\varepsilon_0\,\cancel{2\pi a}\,(a^2+x^2)^{3/2}} = \frac{1}{4\pi\varepsilon_0}\,\frac{x\,q}{(a^2+x^2)^{3/2}}$$

$$x >> a \Rightarrow E \approx \frac{1}{4\pi\varepsilon_0}\,\frac{\cancel{x}\,q}{x^{\cancel{3}\,2}} = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{x^2}$$ (redukuje się do punktu)

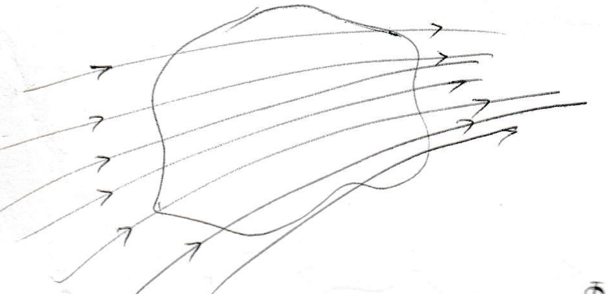
## Strumień pola $\vec{E}$ → prawo Gaussa

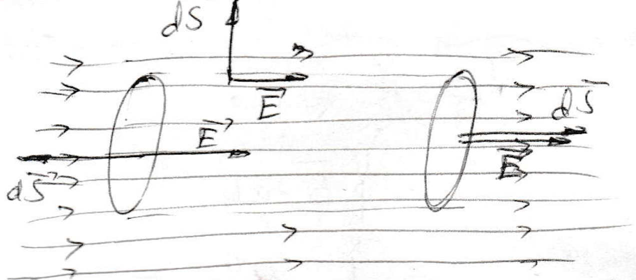
$$\boxed{\Phi_E \approx \sum_i \vec{E}_i \cdot \Delta\vec{S}_i}$$

digresja: $\Phi = \rho\,\vec{v}\cdot\vec{A}$

$$d\Phi_E = |\vec{E}|\,|d\vec{S}|\cos\theta$$

$$\boxed{\Phi_E = \oint \vec{E}\cdot d\vec{S}}$$

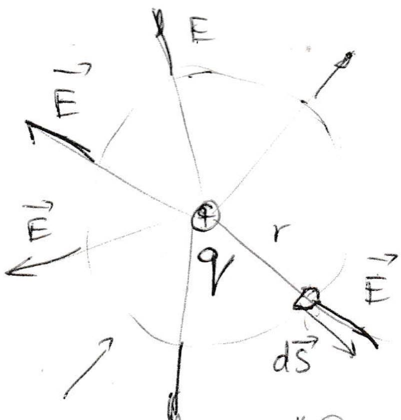




$$\Phi_E = \underset{\text{pob.}}{\int} \underset{0}{\underset{\shortparallel}{\vec{E}\cdot d\vec{S}}} + \underbrace{\underset{\text{pod}}{\int}\vec{E}\cdot d\vec{S} + \underset{\text{pod}}{\int}\vec{E}\cdot d\vec{S}}_{\text{przeciwne znaki}}$$

$$\boxed{\varepsilon_0 \oint \vec{E}\cdot d\vec{S} = q}$$

## Prawo Gaussa a prawo Coulomba



powierzchnia "Gaussowska"

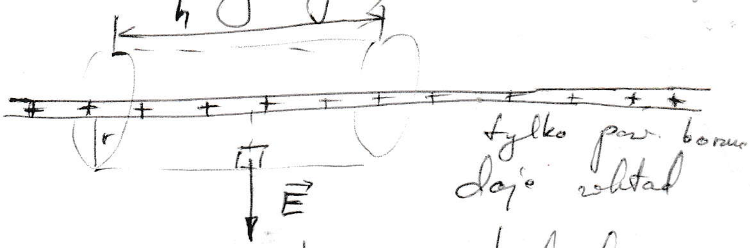
$$\varepsilon_0 \oint \vec{E}\cdot d\vec{S} = \varepsilon_0 \oint_S E\,dS = \varepsilon_0 E \oint dS = q$$

(1) $\vec{E} \parallel d\vec{S}$

(2) $E = \text{const}$ na pow. $S$ } warunki

$$E\,4\pi r^2 \varepsilon_0 = q \Rightarrow \underbrace{E(r) = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{r^2}}_{\text{p. Coulomba}}$$

## Dygresja: Model atomu Thomsona (historyczny!)



tylko pow. boczna daje wkład

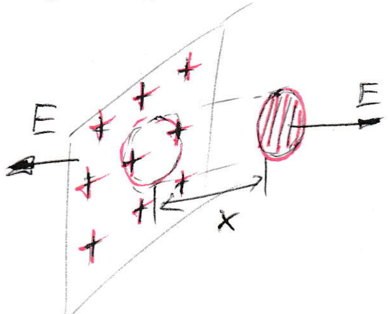
## Przykłady zastosowań p. Gaussa

λ = gęstość liniowa ładunku

$$\varepsilon_0 E(2\pi r \cancel{h}) = \lambda \cancel{h}$$

$$E = \frac{\lambda}{\varepsilon_0 2\pi r}$$

σ = gęstość pow.

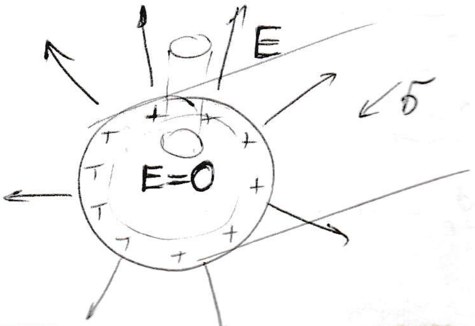


$$\varepsilon_0 \oint E\,dS = \sigma A$$

$$\varepsilon_0 (E\cancel{A} + E\cancel{A}) = \sigma \cancel{A} \quad = \quad E = \frac{\sigma}{2\varepsilon_0}$$

} nie zależy od x

## naładowany przewodnik

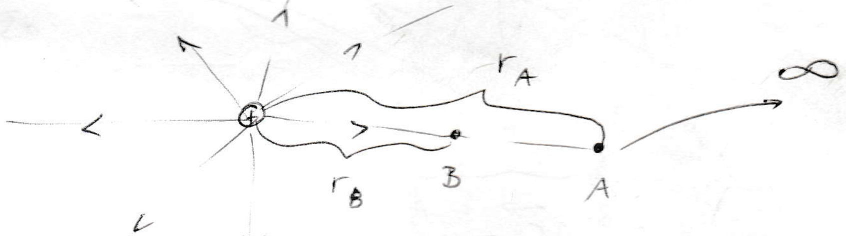


↙σ ⇒ $E = \frac{\sigma}{\varepsilon_0}$

## Układ ładunków punktowych (potencjał)

$$V = -\int \vec{E}\cdot d\vec{l}$$

$$V_B - V_A = -\frac{q}{4\pi\varepsilon_0}\int_{r_A}^{r_B}\frac{dr}{r^2} =$$



$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

$$\boxed{V = \frac{q}{4\pi\varepsilon_0 r}}$$

$$\boxed{V = \sum_n V_n = \frac{1}{4\pi\varepsilon_0}\sum_n \frac{q_n}{r_n}}$$

lub

$$V = \int dV$$

dla dipola

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_1} - \frac{q}{r_2}\right) = \frac{q}{4\pi\varepsilon_0}\frac{(r_2 - r_1)}{r_1 r_2} \cong \frac{q\,l\cos\theta}{4\pi\varepsilon_0 r^2}$$

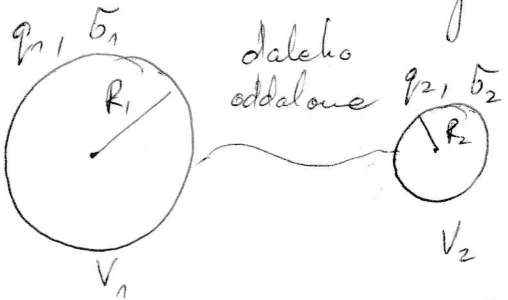
$$r_2 - r_1 \sim l\cos\theta \qquad r_1 r_2 \approx r^2$$

## Energia potencjalna

dwóch ładunków punktowych

$$U = q_2 V = q_2 \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{12}} = \boxed{\frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}}$$

Przykład: Kula naładowana i związek gęstości ładunku z krzywizną



$$V_1 = V_2$$

$$\cancel{\frac{1}{4\pi\varepsilon_0}} \frac{q_1}{R_1} = \cancel{\frac{1}{4\pi\varepsilon_0}} \frac{q_2}{R_2}$$

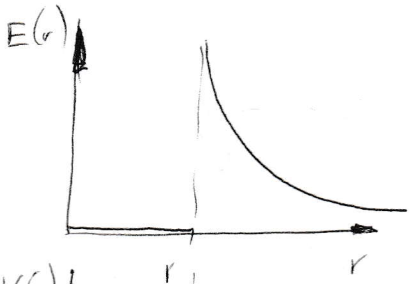
$$\frac{q_1}{q_2} = \frac{R_1}{R_2} \qquad \sigma_1 = \frac{q_1}{4\pi R_1^2} ; \quad \sigma_2 = \frac{q_2}{4\pi R_2^2}$$

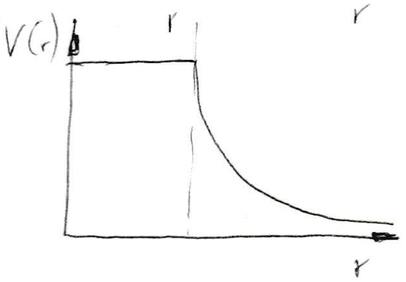
$$\frac{\sigma_1 \cancel{4\pi} R_1^{\cancel{2}}}{\sigma_2 \cancel{4\pi} R_2^{\cancel{2}}} = \frac{\cancel{R_1}}{\cancel{R_2}} \Rightarrow \boxed{\frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1}}$$

$$\boxed{\sigma_1 R_1 = \sigma_2 R_2 = \text{const}}$$

$$V = -\int \vec{E} \cdot d\vec{l}$$

$$\vec{E} = -\frac{dV}{dr}$$





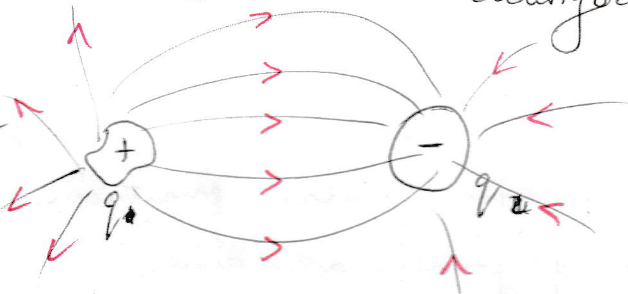
im większy promień (mniejsza krzywizna) tym mniejsza gęstość ładunkowa.

## Kondensatory i dielektryki (jak magazynować energię elektryczną?)

$$\boxed{q = CV}$$

dwa przewodniki odizolowane
różnica potencjałów

$$\boxed{C = \frac{q}{V}} \qquad \left[1F = \frac{1C}{1V}\right]$$



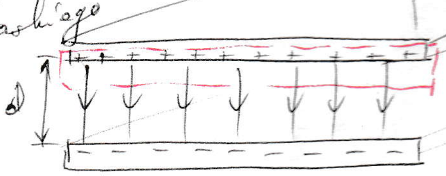
analogia

$$n = \frac{V}{RT} p$$

$$q = C V$$

- pojemność kondensatora płaskiego

$$\boxed{C = \frac{\varepsilon_0 \cancel{E} A}{\cancel{E} d} = \varepsilon_0 \frac{A}{d}}$$



$$\varepsilon_0 E A = q$$

$$V = E d$$

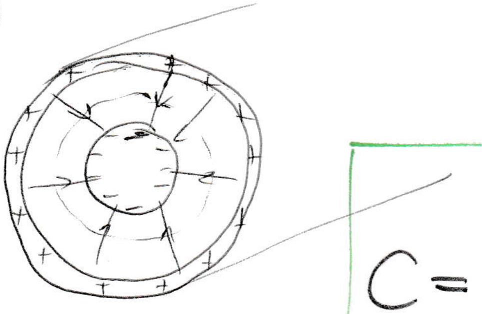
$\varepsilon_0$

$$V = -\int \vec{E} \cdot d\vec{l}$$

- pojemność kond. cylindrycznego

$$\varepsilon_0 E 2\pi r l = q$$

$$V = -\int_a^b \vec{E} \cdot d\vec{l} = \int_a^b E dr = \int_a^b \frac{q}{2\pi\varepsilon_0 l} \frac{dr}{r}$$



$$\boxed{C = \frac{\oint \vec{E} \cdot d\vec{S}}{-\int \vec{E} \cdot d\vec{l}}}$$