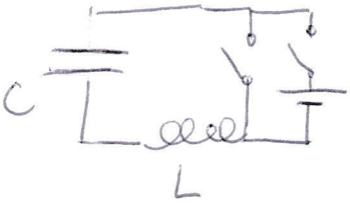
Drgania elektromagnetyczne

Obwód LC (przy zał: $R=0$)

$U_E = \frac{q_m^2}{2C}$ (kondensator) $\qquad U_B = \frac{1}{2}Li^2$ (cewka)

$V_C = \frac{q}{C}$

$V_R = iR$



analogia przemian energii w drganiach mechanicznych

Mechanika	Elektromagnetyzm
$U_p = \frac{1}{2}kx^2$	$U_E = \frac{1}{2}\frac{q^2}{C}$
$U_k = \frac{1}{2}mv^2$	$U_B = \frac{1}{2}Li^2$
$v = \frac{dx}{dt}$	$i = \frac{dq}{dt}$

$q \to x$

$i \to v$

$C \to \frac{1}{k}$

$L \to m$

$\omega = \sqrt{\frac{k}{m}}$

$\omega = \sqrt{\frac{1}{LC}}$

Energetycznie

$\frac{dU}{dt} = 0$

$$\boxed{U = U_E + U_B = \frac{q^2}{2C} + \frac{Li^2}{2} = \text{const}}$$

$$\frac{d}{dt}(\ldots) = \frac{q}{C}\frac{dq}{dt} + Li\frac{di}{dt} = 0 \;/ : i \qquad i = \frac{dq}{dt}, \quad \frac{di}{dt} = \frac{d^2q}{dt^2}$$

$$L\frac{d^2q}{dt^2} + \frac{q}{C}\cancel{\frac{dq}{dt}} = 0$$

$$\boxed{\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0} \;; \quad q = q_m \cos(\omega t + \varphi)$$

przesunięcie fazowe

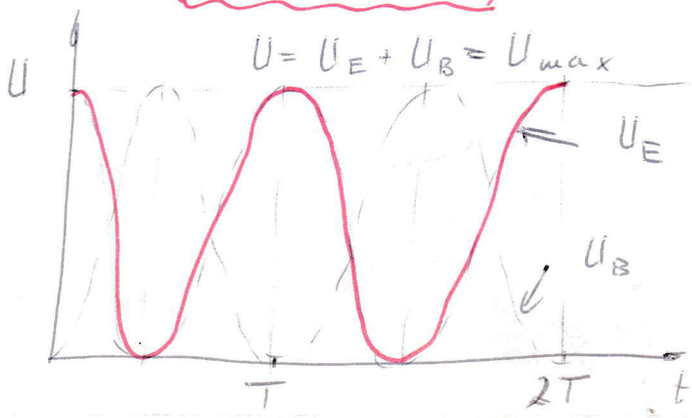
$$\frac{dq}{dt} = i = -\omega q_m \sin(\omega t + \varphi)$$

$$\frac{d^2q}{dt^2} = -\omega^2 q_m \cos(\omega t + \varphi)$$

$$\Rightarrow \boxed{\omega = \frac{1}{\sqrt{LC}}}$$

$$U_B = \frac{1}{2}L q_m^2 \omega^2 \sin^2(\omega t + \varphi)$$

$$U_E = \frac{q_m^2}{2C}\cos^2(\omega t + \varphi)$$



Obwód RLC (tłumienie)

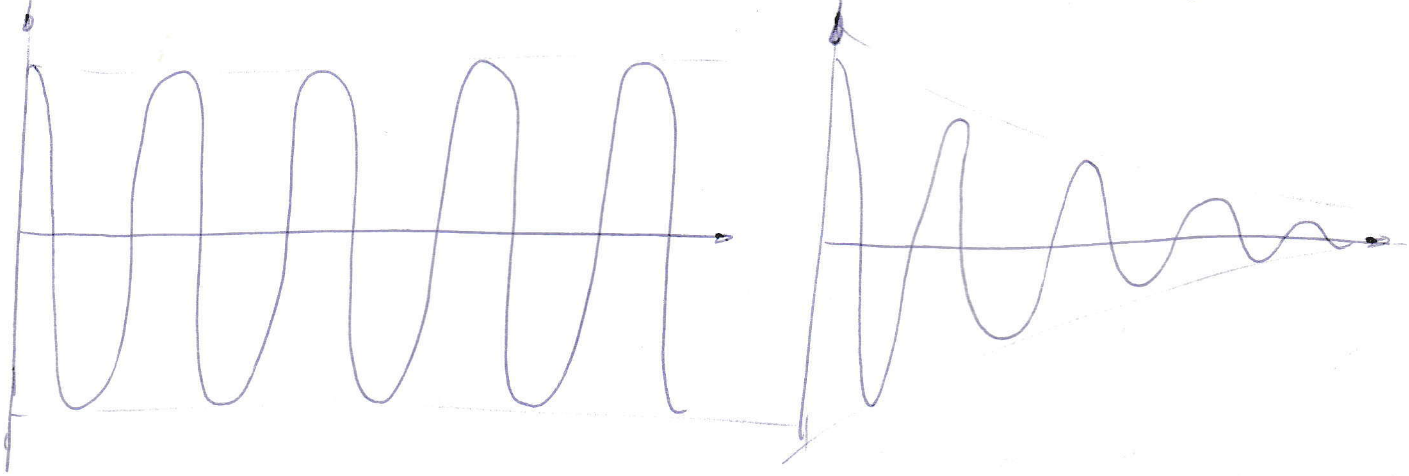
$$U = \frac{1}{2} L i^2 + \frac{1}{2}\frac{q^2}{C} \quad ; \quad \frac{dU}{dt} = -i^2 R$$

$$L\cancel{i}\frac{di}{dt} + \frac{q}{C}\cancel{\frac{dq}{dt}} + i^{\cancel{2}}R = 0$$

analogia

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0 \quad \Big\} \quad m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

$$q = q_m e^{-\frac{Rt}{2L}} \cos(\omega' t) \qquad \omega' = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

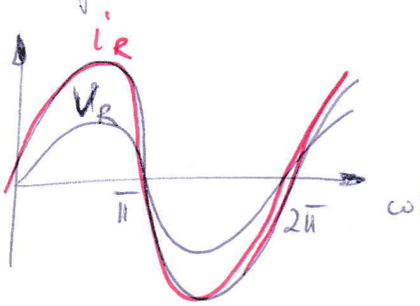


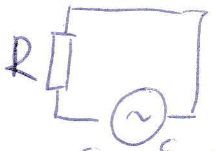
Prądy przemienne (SEM zmienna w czasie)

$\mathcal{E} = \mathcal{E}_m \sin \omega t$  AC (alternating current)

$i = i_m \sin(\omega t - \varphi)$

(1) Opór R





$\mathcal{E} = \mathcal{E}_m \sin \omega t$

$U_R = \mathcal{E}_m \sin \omega t$

$U_R = i_R R \Rightarrow i_R = \frac{\mathcal{E}_m}{R} \sin \omega t$

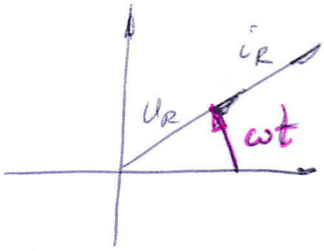
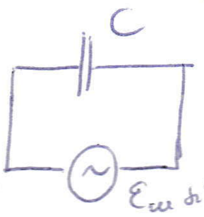


diagram strzałkowy wskazowy

(2) Pojemność C

U_C opóźnia się o $\frac{T}{4}$ do i_C

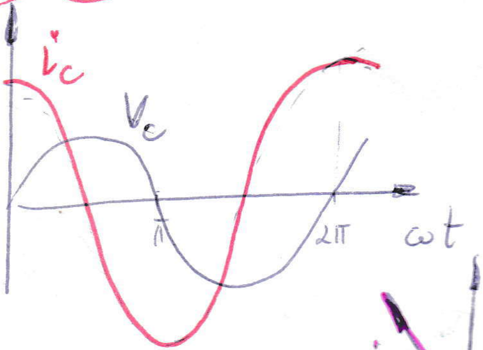


$\mathcal{E}_m \sin \omega t$

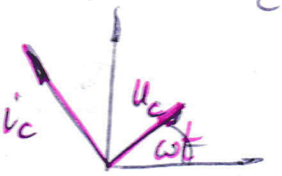
$U_C = \mathcal{E}_m \sin \omega t$

$U_C = \frac{q}{C} \Rightarrow q = C \mathcal{E}_m \sin \omega t$

$i_C = \frac{dq}{dt} = \omega C \mathcal{E}_m \cos \omega t$



$i_C = \frac{\mathcal{E}_m}{X_C} \cos \omega t \Rightarrow \boxed{X_C = \frac{1}{\omega C}}$



oporność pojemnościowa (reaktancja pojemn.)

(3) Indukcyjność L

U_L wyprzedza o $\frac{T}{4}$ i_L

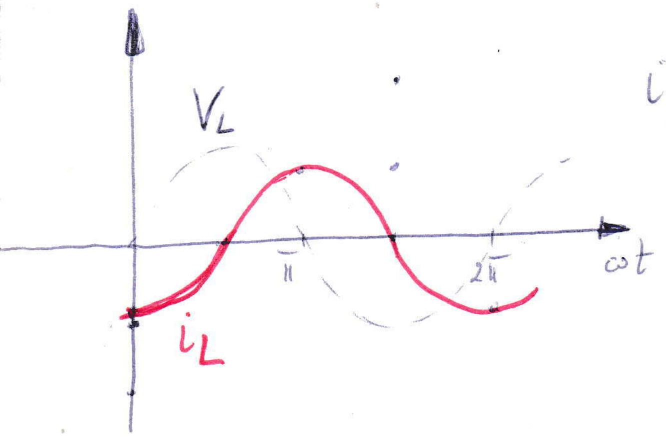
$\mathcal{E}_m \sin \omega t$

$U_L = \mathcal{E}_m \sin \omega t$

$U_L = L \frac{di}{dt}$

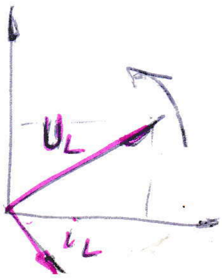
$i = \frac{1}{L} \int \mathcal{E}_m \sin \omega t \, dt =$

$= -\frac{\mathcal{E}_m}{\omega L} \cos \omega t$



$i_L = -\frac{\mathcal{E}_m}{X_L} \cos \omega t \Rightarrow \boxed{X_L = \omega L}$

oporność L (reaktancja indukcyjna)

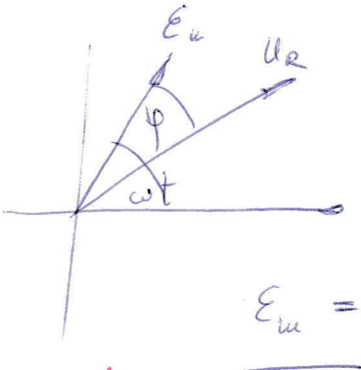


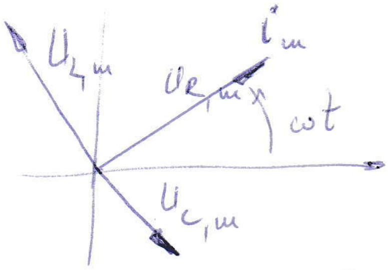
wyprzedza

Obwód RLC

$$\mathcal{E} = \mathcal{E}_m \sin \omega t$$
$$i = i_m \sin(\omega t - \varphi)$$

$$\mathcal{E} = U_R + U_C + U_L$$





$$\mathcal{E}_m = \sqrt{U_{R,m}^2 + (U_{L,m} - U_{C,m})^2} = \sqrt{(i_m R)^2 + (i_m X_L - i_m X_C)^2} =$$

$$= \boxed{i_m \sqrt{R^2 + (X_L - X_C)^2}} \Rightarrow \boxed{i_m = \frac{\mathcal{E}_m}{Z}}$$

zawada (impedancja)

$$\boxed{i_m = \frac{\mathcal{E}_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}}$$

$$\operatorname{tg} \varphi = \frac{U_{L,m} - U_{C,m}}{U_{R,m}} = \frac{X_L - X_C}{R}$$

Moc prądu zmiennego

$$\boxed{P = \frac{\mathcal{E}_m^2}{R}} \quad (\text{stała SEM})$$

$$\boxed{P(t) = \frac{(\mathcal{E}_m \sin \omega t)^2}{R}}$$

chwilowa

tylko opór R

średnia $\overline{P} = \frac{\mathcal{E}_m^2}{R} \overline{\sin^2 \omega t} = \frac{1}{2} \frac{\mathcal{E}_m^2}{R} = \boxed{\left(\frac{\mathcal{E}_m}{\sqrt{2}}\right)^2 \frac{1}{R}}$

$$\boxed{P_{sk} = \frac{\mathcal{E}_{sk}^2}{R}}$$

$\mathcal{E}_{sk}$ – wartość średnia kwadratowa

$$\mathcal{E}_{sk} = \frac{\mathcal{E}_m}{\sqrt{2}} \qquad U_{sk} = \frac{U_m}{\sqrt{2}} \qquad i_{sk} = \frac{i_m}{\sqrt{2}}$$

$$P(t) = \mathcal{E}(t)\, i(t) = \mathcal{E}_m \sin \omega t \; i_m \sin(\omega t - \varphi) = \mathcal{E}_m i_m \sin \omega t$$
$$(\sin \omega t \cos \varphi - \cos \omega t \sin \varphi) = \mathcal{E}_m i_m (\sin^2 \omega t \cos \varphi - \sin \omega t \cos \omega t \sin \varphi)$$

$$\overline{P(t)} = P_{śr} = \frac{1}{2} \mathcal{E}_m i_m \cos \varphi + 0$$

$$\boxed{P_{śr} = \mathcal{E}_{sk}\, i_{sk} \cos \varphi}$$

współczynnik mocy

Rezonans w obwodach prądu zmiennego

$$i_{sk} = \frac{\mathcal{E}_{sk}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = max \iff X_L = X_C \Rightarrow \omega L = \frac{1}{\omega C}$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$i_{sk,max} = \frac{\mathcal{E}_{sk}}{R}$$

zjawisko rezonansu wystąpi $\iff$ $i_{sk} \to \infty$ (dla $R=0$)